Вариант 1

№1.1.

$4x\,dx - 3y\,dy = 3x^2y\,dy - 2xy^2\,dx$

$(4x + 2xy^2)\,dx = (3x^2y + 3y)\,dy$

$2x \cdot (2 + y^2)\,dx = 3y(x^2+1)\,dy$

$\frac{2x}{x^2+1}\,dx = \frac{3y\,dy}{2+y^2}$

$\int \frac{2x\,dx}{x^2+1} = \int \frac{3y\,dy}{2+y^2}$

$t = x^2+1$

$dt = (x^2+1)'\,dx = 2x\,dx$

$p = 2+y^2$

$dp = (2+y^2)'\,dy = 2y\,dy$

$y\,dy = \frac{dp}{2}$

$\int \frac{dt}{t} = \int \frac{3\frac{dp}{2}}{p}$

$\ln|t| = \frac{3}{2}\ln|p| + \ln C.$

$$\ln|t| = \ln\sqrt{p^3 \cdot C}$$

$$t = \sqrt{p^3 \cdot C}$$

$$x^2 + 1 = \sqrt{(2+y^2)^3 \cdot C}$$

4.1 $y' - \frac{y}{x} = x^2 \quad y(1) = 0$

$y = uv$

$y' = u'v + uv'$

$u'v + uv' - \frac{uv}{x} = x^2$

$u'v + u\left(v' - \frac{v}{x}\right) = x^2$

$v' - \frac{v}{x} = 0$

$\frac{dv}{dx} = \frac{v}{x}$

$\frac{dv}{v} = \frac{dx}{x}$

$\int \frac{dv}{v} = \int \frac{dx}{x}$

$\ln v = \ln x$

$v = x.$

$u'v = x^2$

$u'x = x^2$

$u' = x$

$\frac{du}{dx} = x$

$du = x\,dx$

$\int du = \int x\,dx$

$u = \frac{x^2}{2} + C$

$y = uv = \left(\frac{x^2}{2} + C\right) \cdot x = \frac{x^3}{2} + Cx$

$y(1) = \frac{1^3}{2} + C \cdot 1 = 0$

$C = -\frac{1}{2}$

$y = \frac{x^3}{2} - \frac{1}{2}x$

(6.1) $y' + xy = (1+x)e^{-x}y^2$ $\quad y(0) = 1$

$y = uv$ $\quad y' = u'v + uv'$

$$u'v + uv' + xuv = (1+x)e^{-x}(uv)^2$$

$$u'v + u(v' + xv) = (1+x)e^{-x}u^2v^2$$

$$v' + xv = 0$$

$$\frac{dv}{dx} = -xv$$

$$\frac{dv}{v} = -x\,dx$$

$$\int \frac{dv}{v} = -\int x\,dx$$

$$\ln v = -\frac{x^2}{2} + c$$

$$v = e^{-\frac{x^2}{2}}$$

$$u'v = (1+x)e^{-x}u^2v^2$$

$$u' = (1+x)e^{-x}u^2v$$

$$\frac{du}{dx} = (1+x)e^{-x}e^{-\frac{x^2}{2}}u^2$$

$$\frac{du}{u^2} = (1+x)e^{-x-\frac{x^2}{2}}$$

$$\int \frac{du}{u^2} = \int (1+x)\, e^{-x-\frac{x^2}{2}}\, dx$$

$t = \left(-x-\frac{x^2}{2}\right)$

$dt = \left(-x-\frac{x^2}{2}\right)' dx = (-1-x)\,dx =$

$= -(1+x)\,dx$

$(1+x)\,dx = -dt$

$$\int \frac{du}{u^2} = \int e^t (-dt)$$

$$\frac{u^{-1}}{-1} = -e^t \neq C$$

$$\frac{1}{u} = e^t + C$$

$$u = \frac{1}{e^t + C} = \frac{1}{e^{-x-\frac{x^2}{2}} + C}$$

$$y = uv = \frac{1}{e^{-x-\frac{x^2}{2}} + C} e^{-\frac{x^2}{2}}$$

~~$y(1) = \frac{1}{e^{-1-\frac{1}{2}} + C} e^{-\frac{1}{2}} = 0$~~

$y(0) = \frac{1}{e^{0+0}+C} \cdot e^0 = 1$

$1 + C = 1$

$C = 0$

$y = \frac{1}{e^{-x-\frac{x^2}{2}}} \cdot e^{-\frac{x^2}{2}} = e^x$

$\underline{y = e^x}$

№7.1

$3x^2 e^y dx + (x^3 e^y - 1) dy = 0$

$P(x;y) = 3x^2 e^y$

$Q(x;y) = x^3 e^y - 1$

$\frac{\partial P}{\partial y} = (3x^2 e^y)'_y = 3x^2 e^y$

$\frac{\partial Q}{\partial x} = (x^3 e^y - 1)'_x = 3x^2 e^y$

$$\frac{\partial P}{\partial y} = \frac{\partial Q}{\partial x} = 3x^2 e^y$$

$$u(x;y) = \int 3x^2 e^y dx = \frac{3x^3}{3} e^y + C(y) =$$

$$= x^3 e^y + C(y)$$

$$u(x;y) = \int (x^3 e^y - 1) dy =$$

$$= x^3 e^y - y + C(x)$$

$$u(x;y) = x^3 e^y - y + C$$

10.1

$$y''' x \ln x = y''$$

Допускает понижение степени

$$y'' = p \qquad y''' = p'$$

$$p' x \ln x = p$$

$$\frac{dp}{dx} x \ln x = p$$

$$\frac{dp}{p} = \frac{dx}{x \ln x}$$

$$\int \frac{dp}{p} = \int \frac{dx}{x \ln x}$$

$$t = \ln x$$

$$dt = (\ln x)' dx = \frac{1}{x} dx$$

$$\int \frac{dp}{p} = \int \frac{dt}{t}$$

$$\ln|p| = \ln|t| + \ln C_1$$

$$p = t C_1$$

$$y'' = \ln x \, C_1$$

$$y' = \int \ln x \, C_1 \, dx =$$

$$= \left| \begin{array}{l} m = \ln x \quad dn = dx \\ dm = (\ln x)' dx = \frac{1}{x} dx \\ n = \int dx = x \\ \int m \, dn = mn - \int n \, dm \end{array} \right.$$

$y' = C_1(\ln x \cdot x - \int x \cdot \frac{1}{x} dx) =$

$= C_1(x \ln x - \int dx) = C_1(x \ln x - x) + C_2$

$y = \int (C_1(x \ln x - x) + C_2) dx =$

$= C_1 \int x \ln x dx - C_1 \frac{x^2}{2} + C_2 x =$

$m = \ln x; dn = x^2 dx$

$dm = (\ln x)' dx = \frac{1}{x} dx$

$n = \int x dx = \frac{x^2}{2}$

$\int m dn = mn - \int n dm$

$= C_1(\ln x \cdot \frac{x^2}{2} - \int \frac{x^2}{2} \cdot \frac{1}{x} dx) -$

$- C_1 \frac{x^2}{2} + C_2 x = \frac{C_1 x^2 \ln x}{2} -$

$- \frac{C_1}{2} \int x^2 dx - C_1 \frac{x^2}{2} + C_2 x + C_3 =$

$= \frac{C_1 x^2 \ln x}{2} - \frac{3}{4} C_1 x^2 +$

$+ C_2 x + C_3.$

(12.1) $y''' + 3y'' + 2y' = 1 - x^2$

$y''' + 3y'' + 2y' = 0$

$k^3 + 3k^2 + 2k = 0$

$k(k^2 + 3k + 2) = 0$

$k_1 = 0 \quad k^2 + 3k + 2 = 0$

$D = (3)^2 - 4 \cdot 1 \cdot 2 = 1$

$k_2 = \frac{-3+1}{2} = -1$

$k_3 = \frac{-3-1}{2} = -2$

$\overline{y} = C_1 e^{0x} + C_2 e^{-1x} + C_3 e^{-2x}$

общее решение однородного диф. уравн.

$f(x) = 1 - x^2 = e^{0x}(1 - x^2)$

$\alpha = 0$ является корнем хар. ур

$v(x) = xe^{0x}(Ax^2 + Bx + C)$

$v(x) = Ax^3 + Bx^2 + Cx$

$v'(x) = 3Ax^2 + 2Bx + C$

$v''(x) = 6Ax + 2B$

$v'''(x) = 6A$

$v''' + 3v'' + 2v' = 1 - x^2$

$6A + 3(6Ax + 2B) + 2(3Ax^2 + 2Bx + C) = 1 - x^2$

$6A + 18Ax + 6B + 6Ax^2 + 4Bx + 2C = 1 - x^2$

ЧТО

$x^2 \mid 6A = -1 \quad A = -\frac{1}{6}$

$x^1 \mid 18A + 4B = 0 \quad 18\left(-\frac{1}{6}\right) + 4B = 0$

$-3 + 4B = 0$

$B = \frac{3}{4}$

$x^0 \mid 6A + 6B + 2C = 1$

$6\left(-\frac{1}{6}\right) + 6 \cdot \frac{3}{4} + 2C = 1$

$-1 + \frac{9}{2} + 2C = 1$

$$2C = -\frac{5}{2}$$

$$C = -\frac{5}{4}$$

$$v(x) = -\frac{1}{6}x^3 + \frac{3}{4}x^2 - \frac{5}{4}x$$

$$y = u(x) + v(x) = C_1 + C_2 e^{-x} + C_3 e^{-2x} - \frac{1}{6}x^3 + \frac{3}{4}x^2 - \frac{5}{4}x$$

(13.1) $y''' - 4y'' + 5y' - 2y = (16 - 12x)e^{-x}$

линейное неоднородное Φy_{3p}

$$y''' - 4y'' + 5y' - 2y = 0$$

$$k^3 - 4k^2 + 5k - 2 = 0$$

$$k^3 - 4k^2 + 4k + k - 2 = 0$$

$$(k-2)^3 + (k-2) = 0$$

$$(k-2)((k-2)^2 + 1) = 0$$

$k_1 = 2$

$k^2 - 4k + 5 = 0$

$D = (-4)^2 - 4 \cdot 1 \cdot 5 = 16 - 20 = -4$

$k_2 = \frac{4 + 2i}{2} = 2 + i$

$k_3 = \frac{4 - 2i}{2} = 2 - i.$

$\overline{y} = C_1 e^{2x} + e^{2x}(C_2 \cos x + C_3 \sin x)$

$f(x) = (16 - 12x)e^{-x}$

$\alpha = -1$ не является корнем характ. уравн.

$\tilde{y} = e^{-x}(Ax + B)$

$\tilde{y}' = e^{-x}(-Ax - B + A)$

$\tilde{y}'' = e^{-x}(Ax + B - A - A) =$

$= e^{-x}(Ax + B - 2A$

$\tilde{y}''' = e^{-x}(-Ax - B + 2A + A) =$

$= e^{-x}(-Ax - B + 3A)$

$v''' - 4v'' + 5v' - 2v = (16-12x)e^{-x}$

$e^{-x}(-Ax+B+3A) - 4e^{-x}(Ax+B-$

$-2A) + 5e^{-x}(-Ax-B+A) -$

$-2e^{-x}(Ax+B) = (16-12x)e^{-x}$

$-Ax - B + 3A - 4Ax - 4B + 8A -$

$-5Ax - 5B + 5A - 2Ax - 2B =$

$= 16 - 12x$

$-12Ax - 12B + 16A = 16 - 12x$

$x \mid -12A = -12 \quad A = 1$

$x^0 \mid -12B + 16A = 16$

$-12B + 16 \cdot 1 = 16$

$-12B = 0$

$B = 0$

$v(x) = e^{-x}(1x+0) = xe^{-x}$

$y = a(x) + v(x) = C_1 e^{2x} + e^{2x} \cdot$

$\cdot (C_2 \cos x + C_3 \sin x) + xe^{-x}$

№ 16.1

$y'' + \pi^2 y = \frac{\pi^2}{\cos \pi x}$ $\quad y(0) = 3$, $y'(0) = 0$

$y'' + \pi^2 y = 0$

$k^2 + \pi^2 = 0$

$k^2 = -\pi^2$

$k_{1,2} = \pm \pi i$

$y(x) = e^{\alpha x}(C_1 \cos \pi x + C_2 \sin \pi x)$

$= C_1 \cos \pi x + C_2 \sin \pi x$

$f(x) = \frac{\pi^2}{\cos \pi x}$

$y_1 = \cos \pi x \quad y_2 = \sin \pi x$

$$\begin{cases} C_1' y_1 + C_2' y_2 = 0 \\ C_1' y_1' + C_2' y_2' = f(x) \end{cases}$$

$$\begin{cases} C_1' \cos \pi x + C_2' \sin \pi x = 0 \\ C_1' (-\pi \sin \pi x) + C_2' \pi \cos \pi x = \frac{\pi^2}{\cos \pi x} \end{cases}$$

$$C_1' = -C_2' \frac{\sin \pi x}{\cos \pi x}$$

$$-C_2' \frac{\sin \pi x}{\cos \pi x} \cdot (-\pi \sin \pi x) + C_2' \pi \cos \pi x = \frac{\pi^2}{\cos \pi x}$$

$$\pi C_2' \frac{\sin^2 \pi x + \cos^2 \pi x}{\cos \pi x} = \frac{\pi^2}{\cos \pi x}$$

$$C_2' \cdot \frac{1}{\cos \pi x} = \frac{\pi}{\cos \pi x}$$

$$C_2' = \pi$$

$$C_2 = \pi \int dx = \pi x + A$$

$$C_1' = -\pi \frac{\sin \pi x}{\cos \pi x}$$

$$C_1 = -\pi \int \frac{\sin \pi x}{\cos \pi x}\, dx =$$

$$= \frac{\pi}{\pi} \int \frac{d(\cos \pi x)}{\cos \pi x} = \ln|\cos \pi x| + B$$

$$y = (\ln|\cos \pi x| + B)\cos \pi x +$$
$$+ (\pi x + A)\sin \pi x$$

$$y(0) = (\ln \cos 0 + B)\cos 0 +$$
$$+ (\pi \cdot 0 + A)\sin 0 = B = 3$$

$$y' = \frac{1}{\cos \pi x} \cdot (-\pi)\sin \pi x \cos \pi x +$$
$$+ (\ln \cos \pi x + B)(-\pi \sin \pi x) +$$
$$+ \pi \sin \pi x + (\pi x + A)\pi \cos \pi x$$

$$y'(0) = -\pi \sin 0 + (\ln \cos 0 + B) \cdot$$
$$\cdot (-\pi \sin 0) + \pi \sin 0 + (\pi \cdot 0 + A)\pi \cdot \ldots$$

$\cdot \cos 0 = \pi A = 0$

$A = 0$

$y = (\ln|\cos x| + 3)\cos \pi x + \pi x \sin \pi x)$