Кузнецов

№ 14.18

$$\sum_{n=1}^{\infty} \frac{n^5}{(n+1)!}(x+5)^{2n+1}$$

$$u_n = \frac{n^5}{(n+1)!}(x+5)^{2n+1}$$

$$u_{n+1} = \frac{(n+1)^5}{(n+2)!}(x+5)^{2n+3}$$

$$\lim_{n\to\infty}\left|\frac{u_{n+1}}{u_n}\right| = \lim_{n\to\infty}\left|\frac{\frac{(n+1)^5}{(n+2)!}(x+5)^{2n+3}}{\frac{n^5}{(n+1)!}(x+5)^{2n+1}}\right| =$$

$$= \lim_{n\to\infty}\left|\frac{(n+1)^5(x+5)^{2n+3}(n+1)!}{(n+2)!\,n^5(x+5)^{2n+1}}\right| =$$

$$= \lim_{n\to\infty}\left|\frac{(x+5)^2}{(n+2)}\cdot\left(\frac{n+1}{n}\right)^5\right| =$$

$$= |x+5|^2 \lim_{n\to\infty}\left(\frac{1}{n+2}\cdot\left(1+\frac{1}{n}\right)^5\right) =$$

$$= |x+5|^2\cdot\frac{1}{\infty}\cdot\left(1+\frac{1}{\infty}\right)^5 = |x+5|^2\cdot 0\cdot(1+0)^5 =$$

$$= |x+5|^2\cdot 0 = 0 < 1$$ ряд сходится на $(-\infty;+\infty)$

№18.18

$$\ln(1+2x-8x^2) = \ln(1+(2x-8x^2))$$

$$\ln(1+x) = x - \frac{x^2}{2} + \frac{x^3}{3} - \frac{x^4}{4} + \frac{x^5}{5} - \frac{x^6}{6} + \dots$$

$$\ln(1+2x-8x^2) = (2x-8x^2) - \frac{(2x-8x^2)^2}{2} + \frac{(2x-8x^2)^3}{3} - \frac{(2x-8x^2)^4}{4} + \frac{(2x-8x^2)^5}{5} - \frac{(2x-8x^2)^6}{6} + \dots$$

№20.18

$$\int_0^{1,5} \frac{dx}{\sqrt[4]{81+x^4}} = \int_0^{1,5} (81+x^4)^{-\frac{1}{4}} dx$$

$$= \int_0^{1,5} \frac{dx}{\sqrt[4]{81\cdot\left(1+\frac{x^4}{81}\right)}} = \int_0^{1,5} \frac{dx}{3\sqrt[4]{\left(1+\frac{x^4}{81}\right)}} = \frac{1}{3}\int_0^{1,5} \frac{dx}{\sqrt[4]{1+\frac{x^4}{81}}}$$

$$= \frac{1}{3}\int_0^{1,5} \frac{1}{\left(1+\frac{x^4}{81}\right)^{\frac{1}{4}}} dx = \frac{1}{3}\int_0^{1,5} \left(1+\frac{x^4}{81}\right)^{-\frac{1}{4}} dx$$

Используем формулу

$$(1+x)^m = 1 + \frac{m}{1!}x + \frac{m(m-1)}{2!}x^2 +$$

$$+ \frac{m(m-1)(m-2)}{3!}x^3 + \frac{m(m-1)(m-2)(m-3)}{4!}x^4 +$$

$$(1+x)^{-\frac{1}{4}} = 1 + \frac{-\frac{1}{4}}{1!}x + \frac{-\frac{1}{4}\cdot\left(-\frac{1}{4}-1\right)}{2!}x^2 +$$

$$+ \frac{-\frac{1}{4}\left(-\frac{1}{4}-1\right)\left(-\frac{1}{4}-2\right)}{3!}x^3 + \frac{-\frac{1}{4}\left(-\frac{1}{4}-1\right)\left(-\frac{1}{4}-2\right)\left(-\frac{1}{4}-3\right)}{4!}x^4 +$$

$$(1+x)^{-\frac{1}{4}} = 1 - \frac{1}{4}x + \frac{5}{32}x^2 - \frac{15}{128}x^3 +$$

$$+ \frac{195}{2048}x^4 + \dots$$

$$\left(1+\frac{x^4}{81}\right)^{-\frac{1}{4}} = 1 - \frac{1}{4}\left(\frac{x^4}{81}\right) + \frac{5}{32}\cdot\left(\frac{x^4}{81}\right)^2 -$$

$$- \frac{15}{128}\left(\frac{x^4}{81}\right)^3 + \frac{195}{2048}\left(\frac{x^4}{81}\right)^4 + \dots$$

$$\left(1+\frac{x^4}{81}\right)^{-\frac{1}{4}} = 1 - \frac{x^4}{324} + \frac{5}{209952}x^8 -$$

$$\int_0^{1,5} \frac{dx}{\sqrt[4]{81+x^4}} = \frac{1}{3}\int_0^{1,5}\left(1-\frac{x^4}{324}+\frac{5x^8}{209952}-\dots\right)dx =$$

$$= \frac{1}{3}\left(x-\frac{x^5}{324\cdot 5}+\frac{5\cdot x^9}{209952\cdot 9}-\dots\right)\Big|_0^{1,5} =$$

$$= \frac{1}{3}\left(1,5-\frac{1,5^5}{1620}+\frac{5\cdot 1,5^9}{1889568}-\dots\right) =$$

$$= \frac{1}{3}\cdot 0,5 - 0,0047 + 0,00010 =$$

$$= 0,5 - 0,005 = 0,495$$