6.2

$$\sum_{n=1}^{\infty} \frac{1}{n \ln^2(2n+1)}$$

$a_n = \frac{1}{n \ln^2(2n+1)}$

$b_n = \frac{1}{n^2}$ $\sum \frac{1}{n^2}$

$$\lim_{n\to\infty} \frac{a_n}{b_n} = \lim_{n\to\infty} \frac{\frac{1}{n \ln^2(2n+1)}}{\frac{1}{n^2}} =$$

$$= \lim_{n\to\infty} \frac{n}{\ln^2(2n+1)} =$$

$$= \lim_{n\to\infty} \frac{n'}{(\ln^2(2n+1))'} =$$

$$= \lim_{n\to\infty} \frac{1}{2\ln(2n+1)(\ln(2n+1))'} =$$

$$= \lim_{n\to\infty} \frac{2n+1}{2\ln(2n+1)\cdot 2} =$$

$$= \lim_{n \to \infty} \frac{(2n+1)'}{4\left(\ln(2n+1)\right)'} =$$

$$= \lim_{n \to \infty} \frac{2}{4 \cdot \frac{1}{2n+1} \cdot (2n+1)'} =$$

$$= \lim_{n \to \infty} (2n+1) = (2 \cdot \infty + 1) = \infty$$

расходится.

№7.2

$$\sum_{n=1}^{\infty} (-1)^{n+1} \left(\frac{n}{2n+1}\right)^n$$

мажорирующий ряд $\left(\frac{n}{2n+1}\right)^n$

$$\lim_{n \to \infty} \sqrt[n]{|a_n|} = \lim_{n \to \infty} \sqrt[n]{\left(\frac{n}{2n+1}\right)^n} =$$

$$= \lim_{n \to \infty} \frac{n}{2n+1} = \lim_{n \to \infty} \frac{\frac{n}{n}}{\frac{2n}{n} + \frac{1}{n}} =$$

$$= \lim_{n \to \infty} \frac{1}{2 + \frac{1}{n}} = \frac{1}{2 + \frac{1}{\infty}} = \frac{1}{2+0} =$$

$= \frac{1}{2} < 1$ сходится.

№8.2.

$\sum_{n=1}^{\infty} \frac{(-1)^{n+1}}{n!}$ $\alpha = 0{,}01$.

$S = \frac{1}{1} - \frac{1}{1\cdot 2} + \frac{1}{1\cdot 2\cdot 3} - \frac{1}{1\cdot 2\cdot 3\cdot 4} +$

$+ \frac{1}{5!} - \frac{1}{6!} + \frac{1}{7!} - \frac{1}{8!} + \frac{1}{9!} =$

$= 1 - 0{,}5 + 0{,}1667 - 0{,}0417 +$

$+ 0{,}0083 - 0{,}0014 + 0{,}0002 - \ldots \approx$

$\approx 1 - 0{,}5 + 0{,}167 - 0{,}042 + 0{,}008 \approx$

$\approx 0{,}633$

№9.2. Кузнецов.

$\sum_{n=1}^{\infty} \frac{(-1)^{n+1}}{n \ln(1+x)}$

$|u_n| = \frac{1}{n \ln(1+x)}$

$|u_{n+1}| = \frac{1}{(n+1)\ln(1+x)}$

$$\lim_{n\to\infty} \left|\frac{u_{n+1}}{u_n}\right| =$$

$$= \lim_{n\to\infty} \frac{\frac{1}{(n+1)^{\ln(1+x)}}}{\frac{1}{n^{\ln(1+x)}}} =$$

$$= \lim_{n\to\infty} \left(\frac{n}{n+1}\right)^{\ln(1+x)} = 1^{\ln(1+x)} = 1$$

№10.2

$$\sum_{n=1}^{\infty} \frac{(-1)^n}{(n+1)^{5n}} (x-3)^n$$

$$u_n = \frac{(x-3)^n}{(n+1)^{5n}}$$

$$u_{n+1} = \frac{(x-3)^{n+1}}{(n+2)^{5n+5}}$$

$$\lim_{n\to\infty} \left|\frac{u_{n+1}}{u_n}\right| = \lim_{n\to\infty} \left| \frac{\frac{(x-3)^{n+1}}{(n+2)^{5n+5}}}{\frac{(x-3)^n}{(n+1)^{5n}}} \right| =$$

$$= \lim_{n\to\infty} \left| (x-3) \left(\frac{n+1}{n+2}\right)^{5n} \cdot \frac{1}{(n+2)^5} \right| =$$

$$= |x-3| \cdot \lim_{n\to\infty} \left(1 - \frac{1}{n+2}\right)^{\frac{n+2}{-1} \cdot \frac{-1 \cdot 5n}{n+2}} \cdot$$

$$\cdot \frac{1}{(n+2)^5} = |x-3| \cdot e^{\lim_{n\to\infty} \frac{-5n}{n+2}} \cdot \frac{1}{\infty^5} =$$

$$= |x-3| \cdot e^{-\frac{5}{1}} \cdot 0 = 0 < 1$$

ряд сходится при любом x.

№11.2

$$\sum_{n=1}^{\infty} \frac{8^n}{n^2} \sin^{3n} x$$

$$u_n = \frac{8^n}{n^2} \sin^{3n} x$$

$$u_{n+1} = \frac{8^{n+1}}{(n+1)^2} \sin^{3n+3} x$$

$$\lim_{n\to\infty} \left| \frac{u_{n+1}}{u_n} \right| =$$

$$= \lim_{n\to\infty} \left| \frac{\frac{8^{n+1}}{(n+2)^2}\sin^{3n+3}x}{\frac{8^n}{(n+1)^2}\sin^{3n}x} \right|$$

$$= \lim_{n\to\infty} \left| 8 \cdot \left(\frac{n+1}{n+2}\right)^2 \sin^3 x \right| =$$

$$= \left| 8 \lim_{n\to\infty} \left(1 + \frac{1}{n+2}\right)^2 \sin^3 x \right| =$$

$$= \left| 8 \sin^3 x \cdot \left(1 + \frac{1}{\infty}\right)^2 \right| = |8\sin^3 x| < 1.$$

$$-1 < \sin^3 x < 1 \qquad |\sin^3 x| < \frac{1}{8}$$

$$-8 < 8\sin^3 x < 8. \qquad |\sin x| < \frac{1}{2}$$

$$2\pi n - \frac{\pi}{6} \le x \le \frac{\pi}{6} + 2\pi n$$

при $x = \pm \frac{\pi}{6}$

$$\sum_{n=1}^{\infty} \frac{8^n}{n^2} \sin^3\left(\pm\frac{\pi}{6}\right) = \sum_{n=1}^{\infty} \frac{(\pm 1)^n}{n^2}$$

мажорирующий ряд

$\sum \frac{1}{n^2}$ обобщенный гармонический ряд сходится

$\left[-\frac{\pi}{6}+2\pi n; \frac{\pi}{6}+2\pi n\right]$ область сходимости.

№ 14.2

$f(x)=\frac{x^2}{\sqrt{4-5x}}$

$f'(x)=\left(\frac{x^2}{\sqrt{4-5x}}\right)'=\frac{(x^2)'\sqrt{4-5x}-x^2(\sqrt{4-5x})'}{(\sqrt{4-5x})^2}=$

$=\frac{2x\sqrt{4-5x}-x^2\cdot\frac{1}{2\sqrt{4-5x}}\cdot(4-5x)'}{4-5x}=$

$=\frac{2x\sqrt{4-5x}\cdot 2\sqrt{4-5x}-x^2\cdot(-5)}{2\sqrt{(4-5x)^3}}=$

$=\frac{4x(4-5x)+5x^2}{2\sqrt{(4-5x)^3}}=\frac{16x-20x^2+5x^2}{2\sqrt{(4-5x)^3}}=$

$$= \frac{16x - 15x^2}{2\sqrt{(4-5x)^3}}$$

8)

$$x^2 \cdot \frac{1}{\sqrt{4-5x}} = x^2 \cdot \frac{1}{\sqrt{4\left(1-\frac{5x}{4}\right)}} =$$

$$= \frac{x^2}{2} \cdot \frac{1}{\sqrt{1-\frac{5x}{4}}}$$

$$\frac{1}{\sqrt{1-x}} = 1 + \frac{-\frac{1}{2}}{1!}x + \frac{-\frac{1}{2}\left(1-\left(-\frac{1}{2}\right)\right)}{2!}x^2 +$$

$$+ \frac{-\frac{1}{2}\left(1-\left(-\frac{1}{2}\right)\right)\cdot\left(2-\left(-\frac{1}{2}\right)\right)^2}{3!}x^3 + \ldots =$$

$$= 1 - \frac{1}{2}x + \frac{3}{8}x^2 - \frac{5}{16}x^3 + \ldots$$

$$\frac{x^2}{2} \cdot \frac{1}{\sqrt{1-\frac{5x}{4}}} = \frac{x^2}{2} \cdot \left(1 - \frac{1}{2}\cdot\left(-\frac{5x}{4}\right) + \frac{3}{8} \cdot \right.$$

$$\left. \cdot\left(-\frac{5x}{4}\right)^2 - \frac{5}{16}\cdot\left(-\frac{5x}{4}\right)^3 + \ldots\right) = \frac{x^2}{2} \cdot \left(1 + \frac{5x}{8} + \right.$$

$$+\frac{3}{8}\cdot\frac{25}{16}x^2+\frac{5}{16}\cdot\frac{5^3}{4^3}\cdot x^3+\dots)=$$

$$=\frac{x^2}{2}+\frac{5x^3}{16}+\frac{75}{256}x^4+$$

$$+\frac{625}{2048}x^5+\dots$$

№15.2

$$\int_0^{0,1}\sin(100x^2)\,dx$$

$$\sin x=\frac{x}{1!}-\frac{x^3}{3!}+\frac{x^5}{5!}-\frac{x^7}{7!}+\frac{x^9}{9!}-\dots$$

$$\sin(100x^2)=\frac{100x^2}{1!}-\frac{(100x^2)^3}{3!}+$$

$$+\frac{(100x^2)^5}{5!}-\frac{(100x^2)^7}{7!}+\frac{(100x^2)^9}{9!}-\dots$$

$$=100x^2-\frac{100^3\cdot x^6}{3!}+\frac{100^5x^{10}}{5!}-$$

$$-\frac{100^7\cdot x^{14}}{7!}+\frac{100^9x^{18}}{9!}-\dots$$

$$\int_0^{0,1} \sin(100x^2)\,dx =$$

$$= \int_0^{0,1} \left(100x^2 - \frac{100^3 x^6}{3!} + \frac{100^5 x^{10}}{5!} - \right.$$

$$\left. - \frac{100^7 x^{14}}{7!} + \ldots\right) dx =$$

$$= \left(\frac{100 \cdot x^3}{3} - \frac{100^3 x^7}{3!\,7} + \frac{100^5 x^{11}}{5! \cdot 11} - \right.$$

$$\left. - \frac{100^7 \cdot x^{15}}{7! \cdot 12} + \ldots\right)\Bigg|_0^{0,1} = \frac{100 \cdot 0,1^3}{3} -$$

$$- \frac{100^3 \cdot 0,1^7}{3!\,7} + \frac{100^5 \cdot 0,1^{11}}{5!\,11} -$$

$$- \frac{100^7 \cdot 0,1^{15}}{7! \cdot 12} + \ldots = \frac{0,1}{3} - \frac{0,1^7}{3!\,7} +$$

$$+ \frac{0,1}{5! \cdot 11} - \frac{0,1}{7!\,12} + \ldots = 0,0333 -$$

$$- 0,0024 + 0,000076 \approx 0,0333 - 0,0024 = 0,0309$$

№2.2

$$\sum_{n=1}^{\infty} \frac{\operatorname{arctg}^2 n}{n^3}$$

$\sum b_n = \sum \frac{1}{n^3}$ обобщенный гармонич. ряд сходится

$$\lim_{n\to\infty} \frac{a_n}{b_n} = \lim_{n\to\infty} \frac{\frac{\operatorname{arctg}^2 n}{n^3}}{\frac{1}{n^3}} =$$

$$= \lim_{n\to\infty} \operatorname{arctg}^2 n = \operatorname{arctg}^2 \infty =$$

$= \frac{\pi}{2}$ - const

Сходится.

№ 5.2

$$\sum_{n=1}^{\infty} n^4 \left(\frac{2n}{3n+5}\right)^n$$

$$a_n = n^4 \left(\frac{2n}{3n+5}\right)^n$$

$$a_{n+1} = (n+1)^4 \left(\frac{2n+2}{3n+8}\right)^{n+1}$$

Признак Даламбера

$$\lim_{n\to\infty} \frac{u_{n+1}}{u_n} = \lim_{n\to\infty} \frac{(n+1)^4 \left(\frac{2n+2}{3n+8}\right)^{n+1}}{n^4 \left(\frac{2n}{3n+5}\right)^n} =$$

$$= \lim_{n\to\infty} \left(\frac{n+1}{n}\right)^4 \cdot \frac{\left(\frac{2n+2}{3n+8}\right)^n \cdot \left(\frac{2n+2}{3n+8}\right)}{\left(\frac{2n}{3n+5}\right)^n} =$$

$$= \lim_{n\to\infty} \left(1+\frac{1}{n}\right)^4 \cdot \left(\frac{2n+2}{3n+8} \cdot \frac{3n+5}{2n}\right)^n \cdot$$

$$\cdot \left(\frac{2n+2}{3n+8}\right) = \left(1+\frac{1}{\infty}\right)^4 \cdot \lim_{n\to\infty} \left(\frac{6n^2+10n+6n+10}{6n^2+16n}\right)^n \cdot$$

$$\cdot \left(\frac{\frac{2n}{n}+\frac{2}{n}}{\frac{3n}{n}+\frac{8}{n}}\right) = (1+0)^4 \cdot \lim_{n\to\infty} \left(1+\frac{10}{6n^2+16n}\right)^n \cdot$$

$$\cdot \left(\frac{2+\frac{2}{n}}{3+\frac{8}{n}}\right) = 1 \cdot \lim_{n\to\infty} \left(1+\frac{1}{\frac{6n^2+16n}{10}}\right)^{\frac{6n^2+16n}{10} \cdot \frac{10n}{6n^2+16n}} \cdot$$

$\cdot \frac{2}{3} = e^{\lim\limits_{n\to\infty} \frac{10n}{6n^2+16n}}$

$\cdot \frac{2}{3} = e^{\lim\limits_{n\to\infty} \frac{10}{6n+16}} \cdot \frac{2}{3} = e^{\frac{10}{\infty}} \cdot \frac{2}{3} =$

$= e^0 \cdot \frac{2}{3} = \frac{2}{3} < 1$ сходится

№6.2 Кузнецов.

$\sum\limits_{n=1}^{\infty} \frac{1}{n \ln^2(2n+1)}$ интегральный признак коши.

$f(x) = \frac{1}{x \ln^2(2x+1)}$

$\int\limits_1^{\infty} f(x)dx = \int\limits_1^{\infty} \frac{1}{x \ln^2(2x+1)} dx =$

$= \lim\limits_{b\to\infty} \int\limits_1^{b} \frac{1}{x \ln^2(2x+1)} dx =$

$= \left| \begin{array}{l} t = \ln(2x+1) \\ dt = (\ln(2x+1))' dx = \frac{2dx}{2x+1} \\ t_1 = \ln(2\cdot 1+1) = \ln 3 \\ t_2 = \ln(2b+1) \end{array} \right.$