Кузнецов Поле

1.20.

$u = \ln(3 - x^2) + xy^2 z$

$l = -i + 2j - 2k$

$M(1;3;2)$

$\frac{\partial u}{\partial x} = (\ln(3-x^2) + xy^2 z)'_x = \frac{1}{3-x^2}(3-x^2)'_x +$

$+ y^2 z = \frac{-2x}{3-x^2} + y^2 z = \frac{2x}{x^2-3} + y^2 z$

$\frac{\partial u}{\partial y} = (\ln(3-x^2) + xy^2 z)'_y = 2xyz$

$\frac{\partial u}{\partial z} = (\ln(3-x^2) + xy^2 z)'_z = xy^2$

$\frac{\partial u}{\partial x}\Big|_M = \left(\frac{2x}{x^2-3} + y^2 z\right)\Big|_{(1;3;2)} = \frac{2 \cdot 1}{1^2-3} + 3^2 \cdot 2 =$

$= \frac{2}{-2} + 18 = -1 + 18 = 17$

$\frac{\partial u}{\partial y}\Big|_M = (2xyz)\Big|_{(1;3;2)} = 2 \cdot 1 \cdot 3 \cdot 2 = 12$

$\frac{\partial u}{\partial z}\Big|_M = (xy^2)\Big|_{(1;3;2)} = 1 \cdot 3^2 = 9.$

$\vec{l} = -\vec{i} + 2\vec{j} - 2\vec{k} = (-1; 2; -2)$

$a_x = -1 \quad a_y = 2 \quad a_z = -2$

$$\cos\alpha = \frac{a_x}{|\vec{l}|} = \frac{-1}{\sqrt{(-1)^2 + 2^2 + (-2)^2}} = \frac{-1}{\sqrt{9}} = -\frac{1}{3}$$

$$\cos\beta = \frac{a_y}{|\vec{l}|} = \frac{2}{\sqrt{(-1)^2 + 2^2 + (-2)^2}} = \frac{2}{\sqrt{9}} = \frac{2}{3}$$

$$\cos\gamma = \frac{a_z}{|\vec{l}|} = \frac{-2}{\sqrt{(-1)^2 + 2^2 + (-2)^2}} = \frac{-2}{\sqrt{9}} = -\frac{2}{3}$$

$$\left.\frac{\partial u}{\partial \vec{l}}\right|_{M} = \left.\frac{\partial u}{\partial x}\right|_{M} \cdot \cos\alpha + \left.\frac{\partial u}{\partial y}\right|_{M} \cos\beta + \left.\frac{\partial u}{\partial z}\right|_{M} \cos\gamma$$

$$\left.\frac{\partial u}{\partial \vec{l}}\right|_{M} = 17 \cdot \left(-\frac{1}{3}\right) + 12 \cdot \frac{2}{3} + 9 \cdot \left(-\frac{2}{3}\right) =$$

$$= \frac{-17 + 24 - 18}{3} = \frac{-11}{3}.$$

№ 2.20

$u = x^2 - y^2 - 3z^2$

$\vec{a} = \frac{yz^2}{x}$ $\quad M\left(\frac{1}{\sqrt{2}}; \frac{1}{\sqrt{2}}; \frac{1}{\sqrt{3}}\right)$

$\frac{\partial u}{\partial x} = (x^2 - y^2 - 3z^2)'_x = 2x$

$\frac{\partial u}{\partial y} = (x^2 - y^2 - 3z^2)'_y = -2y$

$\frac{\partial u}{\partial z} = (x^2 - y^2 - 3z^2)'_z = -6z$

$\text{grad } u|_M = \frac{\partial u}{\partial x}\Big|_M \vec{i} + \frac{\partial u}{\partial y}\Big|_M \vec{j} + \frac{\partial u}{\partial z}\Big|_M \vec{k}$

$\frac{\partial u}{\partial x}\Big|_M = 2x\Big|_{\left(\frac{1}{\sqrt{2}}; \frac{1}{\sqrt{2}}; \frac{1}{\sqrt{3}}\right)} = 2 \cdot \frac{1}{\sqrt{2}} = \sqrt{2}$

$\frac{\partial u}{\partial y}\Big|_M = -2y\Big|_{\left(\frac{1}{\sqrt{2}}; \frac{1}{\sqrt{2}}; \frac{1}{\sqrt{3}}\right)} = -2 \cdot \frac{1}{\sqrt{2}} = -\sqrt{2}$

$\frac{\partial u}{\partial z}\Big|_M = -6z\Big|_{\left(\frac{1}{\sqrt{2}}; \frac{1}{\sqrt{2}}; \frac{1}{\sqrt{3}}\right)} = -6 \cdot \frac{1}{\sqrt{3}} = -2\sqrt{3}$

$\text{grad } u|_M = \sqrt{2}\vec{i} - \sqrt{2}\vec{j} - 2\sqrt{3}\vec{k}.$

$$\frac{\partial v}{\partial x} = \left(\frac{yz^2}{x}\right)'_x = -\frac{yz^2}{x^2}$$

$$\frac{\partial v}{\partial y} = \left(\frac{yz^2}{x}\right)'_y = \frac{z^2}{x}$$

$$\frac{\partial v}{\partial z} = \left(\frac{yz^2}{x}\right)'_z = \frac{2yz}{x}$$

$$\left.\frac{\partial v}{\partial x}\right|_M = \left.-\frac{yz^2}{x^2}\right|_{\left(\frac{1}{\sqrt{2}};\frac{1}{\sqrt{2}};\frac{1}{\sqrt{3}}\right)} = -\frac{\frac{1}{\sqrt{2}}\cdot\left(\frac{1}{\sqrt{3}}\right)^2}{\left(\frac{1}{\sqrt{2}}\right)^2} =$$

$$= -\frac{\sqrt{2}}{3}$$

$$\left.\frac{\partial v}{\partial y}\right|_M = \left.\frac{z^2}{x}\right|_{\left(\frac{1}{\sqrt{2}};\frac{1}{\sqrt{2}};\frac{1}{\sqrt{3}}\right)} = \frac{\left(\frac{1}{\sqrt{3}}\right)^2}{\frac{1}{\sqrt{2}}} = \frac{\sqrt{2}}{3}$$

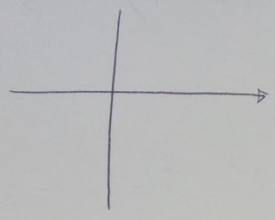
$$\left.\frac{\partial v}{\partial z}\right|_M = \left.\frac{2yz}{x}\right|_{\left(\frac{1}{\sqrt{2}};\frac{1}{\sqrt{2}};\frac{1}{\sqrt{3}}\right)} =$$

$$= \frac{2\cdot\frac{1}{\sqrt{2}}\cdot\frac{1}{\sqrt{3}}}{\frac{1}{\sqrt{2}}} = \frac{2}{\sqrt{3}}$$

$$\operatorname{grad} v|_M = \left(-\frac{\sqrt{2}}{3};\ \frac{\sqrt{2}}{3};\ \frac{2}{\sqrt{3}}\right)$$

3.20

$a = 5y\,i + 7x\,j$ $\quad N = (5y; 7x)$

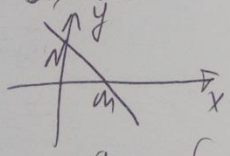


№10.20

$F = (x+y)^2 i - (x^2+y^2) j$

$\#$ dl N: $\quad y = x + 1$

$dy = (x+1)' dx = dx$



$$A = \int (x+y)^2 dx - (x^2+y^2) dy =$$

$$= \int_0 (x + x + 1)^2 dx - (x^2 + (x+1)^2) dx =$$

$$= \int_1^0 (2x+1)^2 - (x^2 + x^2 + 2x + 1) dx =$$

$$= \int_1^0 (4x^2 + 4x + 1 - 2x^2 - 2x - 1) dx =$$

$$= \int_1^0 (2x^2 + 2x)\,dx = \left(\frac{2x^3}{3} + \frac{2x^2}{2}\right)\Big|_1^0 =$$

$$= \frac{2}{3}(0^3 - 1^3) + (0^2 - 1^2) = -\frac{2}{3} - 1 = -\frac{5}{3}$$

$$|\operatorname{grad} u|_M| = \sqrt{(+\sqrt{2})^2 + (-\sqrt{2})^2 + (-2\sqrt{3})^2} =$$

$$= \sqrt{2 + 2 + 12} = \sqrt{16} = 4$$

$$|\operatorname{grad} v|_M| = \sqrt{\left(-\frac{\sqrt{2}}{3}\right)^2 + \left(\frac{\sqrt{2}}{3}\right)^2 + \left(\frac{2}{\sqrt{3}}\right)^2} =$$

$$= \sqrt{\frac{2}{9} + \frac{2}{9} + \frac{4}{3}} = \sqrt{\frac{2+2+12}{9}} = \frac{4}{3}$$

$$\cos\varphi = \frac{(\operatorname{grad} u|_M) \cdot (\operatorname{grad} v|_M)}{|\operatorname{grad} u|_M| \cdot |\operatorname{grad} v|_M|} =$$

$$= \frac{\left(-\sqrt{2}; -\sqrt{2}; -2\sqrt{3}\right)\left(-\frac{\sqrt{2}}{3}; \frac{\sqrt{2}}{3}; \frac{2}{\sqrt{3}}\right)}{4 \cdot \frac{4}{3}} =$$

$$= \frac{\sqrt{2}\cdot\left(-\frac{\sqrt{2}}{3}\right) + \left(-\sqrt{2}\right)\cdot\frac{\sqrt{2}}{3} + \left(-2\sqrt{3}\right)\cdot\frac{2}{\sqrt{3}}}{\frac{16}{3}} =$$

$$= \frac{-\frac{2}{3} - \frac{2}{3} - 4}{\frac{16}{3}} = \frac{-\frac{16}{3}}{\frac{16}{3}} = -1$$

$$\varphi = \arccos(-1) = 180^\circ.$$