№20.20 Кузнецов

$$\int_0^{0,1} \frac{\ln(1+2x)}{x}\,dx \qquad \varepsilon = 0,001$$

$$\ln(1+x) = x - \frac{x^2}{2} + \frac{x^3}{3} - \frac{x^4}{4} + \frac{x^5}{5} - \dots$$

$$\ln(1+2x) = 2x - \frac{(2x)^2}{2} + \frac{(2x)^3}{3} - \frac{(2x)^4}{4} + \frac{(2x)^5}{5} - \dots = 2x - 2x^2 + \frac{8x^3}{3} - 4x^4 + \dots$$

$$\int_0^{0,1} \frac{\ln(1+2x)}{x}\,dx = \int_0^{0,1} \frac{\left(2x - 2x^2 + \frac{8x^3}{3} - 4x^4 + \dots\right)}{x}\,dx$$

$$= \int_0^{0,1} \left(2 - 2x + \frac{8}{3}x^2 - 4x^3 + \dots\right)dx =$$

$$= \left(2x - 2\frac{x^2}{2} + \frac{8}{3}\frac{x^3}{3} - 4\frac{x^4}{4} + \dots\right)\Bigg|_0^{0,1} =$$

$$= 2(0,1-0) - 0,1^2 + \frac{8}{9}\cdot 0,1^3 - 0,1^4 = 0,2 - 0,01 + 0,00088\dots \approx$$

$$\approx 0,001 = 0,19$$

№ 18.20

$$\sum_{n=0}^{\infty} (n^2+4n+3)\,x^{n+1}$$

$$\lim_{n\to\infty} \left|(n^2+4n+3)\,x^{n+1}\right| = 1$$

$$= \infty \cdot x^{n+1} = \infty$$

$$\Sigma S_n = \infty$$

№ 14.20

$$\sum_{n=1}^{\infty} \frac{(x-5)^n}{(n+4)\ln(n+4)}$$

$$u_n = \frac{(x-5)^n}{(n+4)\ln(n+4)}$$

$$u_{n+1} = \frac{(x-5)^{n+1}}{(n+5)\ln(n+5)}$$

$$\lim_{n\to\infty} \left|\frac{u_{n+1}}{u_n}\right| = \lim_{n\to\infty} \left| \frac{\frac{(x-5)^{n+1}}{(n+5)\ln(n+5)}}{\frac{(x-5)^n}{(n+4)\ln(n+4)}} \right| =$$

$$= \lim_{n\to\infty} \left|(x-5)\right| \cdot \left| \frac{(n+4)\ln(n+4)}{(n+5)\ln(n+5)} \right| =$$

$$= \lim_{n\to\infty} |x-5| \cdot \left(\frac{n+5-1}{n+5}\right) \cdot \lim_{n\to\infty} \frac{(\ln(n+4))'}{(\ln(n+5))'} =$$

4.20 Ответ: к/число ряд сходится

$$\sum_{n=2}^{\infty} \frac{n+1}{\left(\sqrt[3]{n}-1\right)\cdot\left(n\sqrt[4]{n^3}-1\right)} =$$

$$\frac{1}{3}^{(4} + 1^{(12} + \frac{3}{4}^{(3} = \frac{4+12+9}{12} = \frac{25}{12}$$

$$1 - \frac{25}{12} = -\frac{13}{12}$$

$$\sum b_n = \sum_{n=2}^{\infty} \frac{1}{n^{\frac{13}{12}}} \qquad p = \frac{13}{12} > 1$$

ряд сходится.

$$\lim_{n\to\infty} \frac{a_n}{b_n} = \lim_{n\to\infty} \frac{\frac{n+1}{\left(\sqrt[3]{n}-1\right)\left(n\sqrt[4]{n^3}-1\right)}}{\frac{1}{n^{\frac{13}{12}}}} =$$

$$= \lim_{n\to\infty} \frac{(n+1)\,n^{\frac{13}{12}}}{\left(\sqrt[3]{n}-1\right)\left(n\sqrt[4]{n^3}-1\right)} =$$

$$= \lim_{n\to\infty} \frac{\frac{(n+1)\,n^{\frac{13}{12}}}{n^{\frac{25}{12}}}}{\left(\frac{\sqrt[3]{n}}{\sqrt[3]{n}} - \frac{1}{\sqrt[3]{n}}\right)\left(\frac{n\sqrt[4]{n^3}}{n\sqrt[4]{n^3}} - \frac{1}{n\sqrt[4]{n^3}}\right)}$$

$$\lim_{n\to\infty} \frac{1+\frac{1}{n}}{\left(1-\frac{1}{\sqrt[3]{n}}\right)\left(1-\frac{1}{n\sqrt[4]{n^3}}\right)} = \frac{1+0}{(1-0)(1-0)} = 1$$

$= \lim_{n\to\infty} |x-5| \cdot \left(1 - \frac{1}{n+5}\right) \cdot$

$\cdot \lim_{n\to\infty} \frac{\frac{1}{n+4}}{\frac{1}{n+5}} = \lim_{n\to\infty} |x-5| \cdot (1-0) \cdot$

$\cdot \lim_{n\to\infty} \frac{n+5}{n+4} = \lim_{n\to\infty} |x-5| \cdot 1 \cdot$

$\cdot \lim_{n\to\infty} \left(1 + \frac{1}{n+4}\right) = |x-5| \cdot$

$\cdot (1+0) = |x-5|$

Ряд сходится при

$|x-5| < 1$

$-1 < x-5 < 1$

$4 < x < 6$

$(4,6)$ – интервал сходимости

~~найдем~~ проверим на сходимость на концах интервала

При $x = 4$

$\sum_{n=1}^{\infty} \frac{(4-5)^n}{(n+4)\ln(n+4)} = \sum \frac{(-1)^n}{(n+4)\ln(n+4)}$

$\lim_{n\to\infty} |a_n| = \lim_{n\to\infty} \frac{1}{(n+4)\ln(n+4)} =$

$= \frac{1}{\infty} = 0$

$|a_1| = \frac{1}{5\ln 5} = 0{,}1243$

$|a_2| = \frac{1}{6\ln 6} = 0{,}109$

$|a_3| = \frac{1}{7\ln 7} = 0{,}07$

$0{,}12 > 0{,}09 > 0{,}07 > \ldots$

ряд сходится

при $x = 6$ $\sum_{n=1}^{\infty} \frac{1}{(n+4)\ln(n+4)}$

~~$\lim_{n\to\infty} |a_n| = \lim_{n\to\infty}$~~ 1

~~$\int_1^{\infty} f(x)\,dx = \lim_{a\to\infty} \int_1^a \frac{dx}{(x+4)\ln(x+4)} =$~~

$= \lim_{a\to\infty} \int_1^a \frac{dt}{t}$ | $t = \ln(x+4)$, $dt = \frac{dx}{x+4}$, $t_1 = \ln(1+4) = \ln 5$

$t_2 = \ln(a+4)$

$$= \lim_{a\to\infty} \int_{\ln 5}^{\ln(a+4)} \frac{dt}{t} = \lim_{a\to\infty} \ln t \Big|_{\ln 5}^{\ln a+4} =$$

$$= \lim_{a\to\infty} \ln\ln(a+4) - \ln\ln 5 =$$

$$= \ln\ln\infty - \ln\ln 5 = \infty$$

$\Rightarrow$ рядрасх.

$[4;6)$ обл. сход.

№ 11.20

$$\sum_{n=1}^{\infty} \frac{\sqrt{n}}{n^{x^2-1}} \qquad u_n = \frac{\sqrt{n}}{n^{x^2-1}}$$

$$u_{n+1} = \frac{\sqrt{n+1}}{(n+1)^{x^2-1}}$$

$$\lim_{n\to\infty} \left|\frac{u_{n+1}}{u_n}\right| = \lim_{n\to\infty} \frac{\frac{\sqrt{n+1}}{(n+1)^{x^2-1}}}{\frac{\sqrt{n}}{n^{x^2-1}}} =$$

$=$

$$= \lim_{n\to\infty} \sqrt{\frac{n+1}{n}} \cdot \lim_{n\to\infty} \frac{n^{x^2-1}}{(n+1)^{x^2-1}} =$$

$$= \lim_{n\to\infty} \sqrt{1+\frac{1}{n}} \cdot \lim_{n\to\infty} \left(\frac{n}{n+1}\right)^{x^2-1} =$$

$$= \sqrt{1+0} \lim_{n\to\infty} \left(\frac{n+1-1}{n+1}\right)^{x^2-1} =$$

$$= 1 \cdot \lim_{n\to\infty} \left(1 - \frac{1}{n+1}\right)^{x^2-1} = 1^{(x^2-1)} = 1$$

$$\lim_{n\to\infty} \frac{\sqrt{n}}{n^{x^2-1}} = \infty$$

$$\mu = \iint_D \rho(x;y)\,dx\,dy$$

Перейдем к ~~полярным~~ полярным координатам

$$\mu = \iint \frac{y}{x^3}\,dx\,dy = \int_1^3 \rho\,d\rho \int_0^{2\pi} \frac{4\rho\sin\varphi}{(\rho\cos\varphi)^3}\,d\varphi =$$

$$= 4\int_1^3 \frac{1}{\rho}\,d\rho \cdot \int_0^{2\pi} \frac{\sin\varphi}{\cos^3\varphi}\,d\varphi = 4\ln\rho\Big|_1^3 \cdot$$

$$\cdot 2\,\frac{\cos^{-2}\varphi}{-2}\Big|_0^{2\pi} = 4(\ln 3 - \ln 1)\cdot(-2)\cdot\left(\frac{1}{\cos 2\pi} - \frac{1}{\cos 0}\right) =$$

$$= -4\ln 3\cdot\left(\frac{1}{-1} - \frac{1}{+1}\right) =$$

$$= 8\ln 3$$