Вар № 9

1.9) $\sum_{n=2}^{\infty} \frac{1}{n^2+n-2}$.

$n^2+n-2=0$

$D = 1^2 - 4 \cdot 1 \cdot (-2) = 9$

$n_1 = \frac{-1+3}{2} = 1 \quad n_2 = \frac{-1-3}{2} = -2$

$$\frac{1}{n^2+n-2} = \frac{1}{(n-1)(n+2)} = \frac{A}{n-1} + \frac{B}{n+2} =$$

$$= \frac{An+2A+Bn-B}{(n-1)(n+2)} = \frac{n(A+B)+2A-B}{(n-1)(n+2)}$$

$n \mid A+B=0$

$n^0 \mid 2A-B=1$

$3A = 1$

$A = \frac{1}{3}$

$B = -A = -\frac{1}{3}$

$$\frac{1}{(n^2+n-2)} = \frac{\frac{1}{3}}{n-1} - \frac{\frac{1}{3}}{n+2} =$$

$$= \frac{1}{3}\left(\frac{1}{n-1} - \frac{1}{n+2}\right)$$

$$\sum_{n=2}^{\infty} \frac{1}{n^2+n-2} = \frac{1}{3}\left(\frac{1}{1} - \frac{1}{4} + \frac{1}{2} - \frac{1}{5} + \right.$$

$$+\frac{1}{3}-\frac{1}{6}+\frac{1}{4}-\frac{1}{7}+\frac{1}{5}-\frac{1}{8}+\ldots$$
$$+\frac{1}{n-5}-\frac{1}{n-2}+\frac{1}{n-4}-\frac{1}{n-1}+$$
$$+\frac{1}{n-3}-\frac{1}{n}+\frac{1}{n-2}-\frac{1}{n+1}+$$
$$+\frac{1}{n-1}+\frac{1}{n+2}\Big)=\frac{1}{3}\Big(1+\frac{1}{2}+\frac{1}{3}+$$
$$-\frac{1}{n}-\frac{1}{n+1}-\frac{1}{n+2}\Big)$$

$$S=\lim_{n\to\infty} S_n=\lim_{n\to\infty}\frac{1}{3}\Big(1+\frac{1}{2}+\frac{1}{3}-$$
$$-\frac{1}{n}-\frac{1}{n+1}-\frac{1}{n+2}\Big)=\frac{1}{3}\Big(1+\frac{1}{2}+$$
$$+\frac{1}{3}-0-0-0\Big)=\frac{1}{3}\cdot\frac{6+3+1}{6}=$$
$$=\frac{1}{3}\cdot\frac{10}{6}=\frac{5}{9}.$$

2.9

$$\sum_{n=1}^{\infty}\frac{3n-2}{n(n+1)(n+2)}.$$

$$= \left(\frac{25}{3}\cdot 0{,}2^3 - \frac{25^3}{3!7}\cdot 0{,}2^7 + \frac{25^5}{5!\cdot 11}\cdot 0{,}2^{11} - \right.$$

$$\left. - \frac{25^7}{7!\,15}\cdot 0{,}2^{15} + \frac{25^9}{9!\,19}\cdot 0{,}2^{19} - \ldots\right) =$$

$$= 0{,}0667 - 0{,}0048 + 0{,}00015 -$$

$$- 0{,}0000026 + \ldots \approx 0{,}067 -$$

$$- 0{,}005 \approx 0{,}062.$$

№ 19.9.

$(x-1)\sin 5x$

$$\sin x = x - \frac{x^3}{3!} + \frac{x^5}{5!} - \frac{x^7}{7!} + \ldots$$

$$\sin 5x = 5x - \frac{(5x)^3}{3!} + \frac{(5x)^5}{5!} - \frac{(5x)^7}{7!} + \ldots$$

$$(x-1)\sin 5x = (x-1)\cdot\left(5x - \frac{5^3x^3}{3!} + \right.$$

$$\left. + \frac{5^5x^5}{5!} - \frac{5^7x^7}{7!} + \ldots\right) = 5x^2 - \frac{5^3x^4}{3!} +$$

$$+ \frac{5^5x^6}{5!} - \frac{5^7x^8}{7!} + \ldots - 5x + \frac{5^3x^3}{3!} -$$

$$- \frac{5^5x^5}{5!} + \frac{5^7x^7}{7!} - \ldots$$

$$\frac{3n-2}{n(n+1)(n+2)} = \frac{A}{n} + \frac{B}{n+1} + \frac{C}{n+2} =$$

$$= \frac{A(n+1)(n+2) + Bn(n+2) + Cn(n+1)}{n(n+1)(n+2)} =$$

$$= \frac{An^2 + 3An + 2A + Bn^2 + 2Bn + Cn^2 + Cn}{n(n+1)(n+2)} =$$

$$= \frac{n^2(A+B+C) + n(3A+2B+C) + 2A}{n(n+1)(n+2)}$$

$$\begin{array}{l|l|l} n^2 & A+B+C=0 & \cdot(-1) \\ n^1 & 3A+2B+C=3 & 2A+B=3 \\ n^0 & 2A=-2 \quad A=-1 & 2\cdot(-1)+B=3 \\ & & \underline{B=5} \end{array}$$

$$-1+5+C=0$$

$$\underline{C=-4}.$$

$$\frac{3n-2}{n(n+1)(n+2)} = \frac{-1}{n} + \frac{5}{n+1} + \frac{-4}{n+2}.$$

$$\sum_{n=1}^{\infty} \frac{3n-2}{n(n+1)(n+2)} = -\frac{1}{1} + \frac{5}{2} - \frac{4}{3} +$$

$$-\frac{1}{2} + \frac{5}{3} - \frac{4}{4} + \frac{1}{3} + \frac{5}{4} - \frac{4}{5} - \frac{1}{4} +$$

$$+\frac{5}{5}-\frac{4}{6}+-\frac{1}{5}+\frac{5}{6}-\frac{4}{7}-$$
$$-\frac{1}{6}+\frac{5}{7}-\frac{4}{8}+\ldots\ldots-$$
$$-\frac{1}{n-3}+\frac{5}{n-2}-\frac{4}{n-1}-\frac{1}{n-2}+$$
$$+\frac{5}{n-1}-\frac{4}{n}-\frac{1}{n-1}+\frac{5}{n}-$$
$$-\frac{4}{n+1}-\frac{1}{n}+\frac{5}{n+1}-\frac{4}{n+2}=$$
$$=-1+\frac{5}{2}-\frac{1}{2}+\frac{1}{n+1}-\frac{4}{n+2}=$$
$$=-1+2+\frac{1}{n+1}-\frac{4}{n+2}=$$
$$=1+\frac{1}{n+1}-\frac{4}{n+2}$$
$$S=\lim_{n\to\infty} S_n=\lim_{n\to\infty}\left(1+\frac{1}{n+1}-\frac{4}{n+2}\right)=1+0-0=1.$$

№ 3.9.

$$\sum_{n=1}^{\infty} \frac{\sin^2 n}{n^2+1}$$

используем признак сравнения рядов.

$-1 \le \sin n \le 1$

$0 \le \sin^2 n \le 1$

$$\frac{\sin^2 n}{n^2+1} \le \frac{1}{n^2+1} \le \frac{1}{n^2}$$

$\sum \frac{1}{n^2}$ обобщенный гармонический ряд $p=2>1$ ряд сходится $\Rightarrow \sum \frac{\sin^2 n}{n^2+1}$ сходится

№ 4.9.

$$\sum_{n=1}^{\infty} \frac{1}{n-\cos^2 6n}$$

Используем II признак сравнения рядов. $\sum \frac{1}{n}$ гарм. ряд.

$$\lim_{n\to\infty} \frac{a_n}{b_n} =$$

$$= \lim_{n \to \infty} \frac{1}{\frac{n - \cos^2 6n}{\frac{1}{n}}} =$$

$$= \lim_{n \to \infty} \frac{n}{n - \cos^2 6n} =$$

$$= \lim_{n \to \infty} \frac{(n)'}{(n - \cos^2 6n)'} = \lim_{n \to \infty} \frac{1}{1 + 2\cos 6n \sin 6n \cdot 6}$$

$$= \lim_{n \to \infty} \frac{1}{1 + 6 \sin 6n} = \frac{1}{1 + \alpha}$$ Конец

мое число

ряд расходится

$\alpha \sim 5.9$

$$\sum_{n=1}^{\infty} \frac{n!}{(2n)!} \operatorname{tg} \frac{1}{5^n}$$

Применим признак Даламбера

$$a_n = \frac{n!}{(2n)!} \operatorname{tg} \frac{1}{5^n} \quad a_{n+1} = \frac{(n+1)!}{(2n+2)!} \operatorname{tg} \frac{1}{5^{n+1}}$$

$$\lim_{n\to\infty}\frac{a_{n+1}}{a_n}=\lim_{n\to\infty}\frac{\frac{(n+1)!}{(2n+2)!}\operatorname{tg}\frac{1}{5^{n+1}}}{\frac{n!}{(2n)!}\operatorname{tg}\frac{1}{5^n}}=$$

$$=\lim_{n\to\infty}\frac{(n+1)!}{n!}\cdot\frac{(2n)!}{(2n+2)!}\cdot\frac{\operatorname{tg}\frac{1}{5^{n+1}}}{\operatorname{tg}\frac{1}{5^n}}=$$

$$=\left|\begin{array}{l}\text{при } n\to\infty\ \operatorname{tg}\frac{1}{5^{n+1}}\sim\frac{1}{5^{n+1}}\\ \operatorname{tg}\frac{1}{5^n}\sim\frac{1}{5^n}\end{array}\right|=$$

$$=\lim_{n\to\infty}\frac{n+1}{1}\cdot\frac{1}{(2n+2)}\cdot\frac{\frac{1}{5^{n+1}}}{\frac{1}{5^n}}=$$

$$=\lim_{n\to\infty}\frac{1}{2}\cdot\frac{1}{5}=\frac{1}{10}<1$$

ряд сходится.

№6.9

$$\sum_{n=1}^{\infty}n\arcsin^n\frac{\pi}{4n}$$

Признак Даламбера.

$$a_n = n \arcsin^n \frac{\pi}{4n}$$

$$a_{n+1} = (n+1) \arcsin^{n+1} \frac{\pi}{4(n+1)}$$

$$\lim_{n\to\infty} \frac{a_{n+1}}{a_n} = \lim_{n\to\infty} \frac{(n+1) \arcsin^{n+1} \frac{\pi}{4(n+1)}}{n \arcsin^n \frac{\pi}{4n}} =$$

при $n \to \infty$ $\arcsin \frac{\pi}{4(n+1)} \sim \frac{\pi}{4(n+1)}$ $\arcsin \frac{\pi}{4n} \sim \frac{\pi}{4n}$

$$= \lim_{n\to\infty} \left(1 + \frac{1}{n}\right) \cdot \frac{\left(\frac{\pi}{4(n+1)}\right)^{n+1}}{\left(\frac{\pi}{4n}\right)^n} =$$

$$= (1+0) \cdot \lim_{n\to\infty} \left(\frac{\frac{\pi}{4(n+1)}}{\frac{\pi}{4n}}\right)^n \cdot \frac{\pi}{4(n+1)} =$$

$$= 1 \cdot \lim_{n\to\infty} \left(\frac{n}{n+1}\right)^n \cdot \frac{\pi}{4(n+1)} =$$

$$= \lim_{n\to\infty} \left(1 + \frac{-1}{n+1}\right)^n \cdot \frac{\pi}{4(n+1)} =$$

$$= \lim_{n\to\infty} \left(1 + \frac{1}{\frac{n+1}{-1}}\right)^{\frac{n+1}{-1}\cdot\frac{-1}{n+1}\cdot n \cdot \frac{\pi}{4(n+1)}} =$$

$$= e^{\lim_{n\to\infty} \frac{-n}{n+1} \cdot \frac{\pi}{4(n+1)}} = e^{\lim_{n\to\infty} \frac{-1}{1+\frac{1}{n}} \cdot \frac{\pi}{4(n+1)}} = e^{-1\left(\frac{\pi}{\infty}\right)} = \frac{1}{e}\cdot 0 = 0 < 1$$

ряд сходится.

(7.9) $\sum_{n=1}^{\infty} \frac{1}{(2n-1)\ln 2n}$.

$$a_n = \frac{1}{(2n-1)\ln 2n}$$

$\sum_{n=1}^{\infty} b_n = \sum \frac{1}{n \ln n}$ ряд сходится

$$\lim_{n\to\infty} \frac{a_n}{b_n} = \lim_{n\to\infty} \frac{\frac{1}{(2n-1)\ln 2n}}{\frac{1}{n\ln n}} =$$

$$= \lim_{n\to\infty} \frac{n\ln n}{(2n-1)\ln 2n} =$$

$$= \lim_{n\to\infty} \frac{n}{2n-1} \cdot \lim_{n\to\infty} \frac{\ln n}{\ln 2n}$$

$$= \lim_{n\to\infty} \frac{\frac{n}{n}}{\frac{2n}{n} - \frac{1}{n}} \lim_{n\to\infty} \frac{(\ln n)'}{(\ln 2n)'} =$$

$$= \lim_{n\to\infty} \frac{1}{2 - \frac{1}{n}} \lim_{n\to\infty} \frac{\frac{1}{n}}{\frac{1}{2n} \cdot 2} =$$

$$= \lim_{n\to\infty} \frac{1}{2 - \frac{1}{n}} \cdot 1 = \frac{1}{2-0} = \frac{1}{2}$$

конечное число

ряд сходится

(8.9) $$\sum_{n=1}^{\infty} \frac{(-1)^n \sin \frac{\pi}{2\sqrt{n}}}{\sqrt{3n+1}}$$

Построим мажорирующий ряд.

$$\sum \frac{\sin \frac{\pi}{2\sqrt{n}}}{\sqrt{3n+1}}$$

$$-1 \le \sin \frac{\pi}{2\sqrt{n}} \le 1$$

$$\frac{\sin\frac{\pi}{2\sqrt{n}}}{\sqrt{3n+1}} \sim \frac{1}{\sqrt{3n+1}} \sim \frac{1}{\sqrt{n}}$$

$\sum \frac{1}{\sqrt{n}}$ — обобщенный гарм. ряд расходится

$$\lim_{n\to\infty} \frac{\frac{\sin\frac{\pi}{2\sqrt{n}}}{\sqrt{3n+1}}}{\frac{1}{\sqrt{n}}} =$$

$$= \lim_{n\to\infty} \sin\frac{\pi}{2\sqrt{n}} \cdot \frac{n}{\sqrt{3n+1}} =$$

$$= \lim_{n\to\infty} \frac{\pi}{2\sqrt{n}} \cdot \frac{n}{\sqrt{3n+1}} =$$

$$= \lim_{n\to\infty} \frac{\pi\sqrt{n}}{2\sqrt{3n+1}} = \lim_{n\to\infty} \frac{\pi}{2\sqrt{3+\frac{1}{n}}} =$$

$$= \frac{\pi}{2\sqrt{3+0}} = \frac{\pi}{2\sqrt{3}}$$ конечное число

ряды расходятся $\Rightarrow$ ряд из модулей расход.

Проверим на сходимость ряд по признаку Лейбница.

$$\lim_{n\to\infty} |a_n| = \lim_{n\to\infty} \frac{\sin\frac{\pi}{2\sqrt{n}}}{\sqrt{3n+1}} =$$

$$= \lim_{n\to\infty} \frac{\frac{\pi}{2\sqrt{n}}}{\sqrt{3n+1}} = \frac{0}{\sqrt{\infty}} = 0$$

$$|a_1| = \frac{\sin\frac{\pi}{2\sqrt{1}}}{\sqrt{3\cdot 1+1}} = \frac{\sin\frac{\pi}{2}}{2} =$$

$$= 0{,}5$$

$$|a_2| = \frac{\sin\frac{\pi}{2\sqrt{2}}}{\sqrt{5}} = 0{,}40$$

$$|a_3| = \frac{\sin\frac{\pi}{2\sqrt{3}}}{\sqrt{6}} = 0{,}32$$

$0{,}5 > 0{,}40 > 0{,}32 > \dots$

ряд сходится по признаку Лейбница

№9.9

$$\sum_{n=1}^{\infty} \frac{(-1)^n n}{(2n-1)^2(2n+1)^2} \quad \alpha = 0{,}001.$$

$$S = \lim_{n\to\infty} S_n = \lim_{n\to\infty} \left(\frac{-1}{1 \cdot 9} + \frac{2}{9 \cdot 25} - \frac{3}{25 \cdot 49} + \frac{4}{49 \cdot 81} - \frac{5}{81 \cdot 121} + \frac{6}{121 \cdot 169} - \dots \right) = -0{,}1111 +$$

$$+ 0{,}0089 - 0{,}0024 + 0{,}0010 -$$

$$- 0{,}0005 + 0{,}0002 - \dots \approx$$

$$\approx -0{,}111 + 0{,}009 - 0{,}002 + 0{,}001 -$$

$$- 0{,}001 \approx -0{,}104$$

№ 11.9.

$$\sum_{n=1}^{\infty} \frac{1}{n+3}\left(\frac{1+x}{1-x}\right)^n$$

$$u_n = \frac{1}{n+3}\left(\frac{1+x}{1-x}\right)^n$$

$$u_{n+1} = \frac{1}{n+3+1} \cdot \left(\frac{1+x}{1-x}\right)^{n+1}$$

$$\lim_{n\to\infty} \frac{u_{n+1}}{u_n} = \lim_{n\to\infty} \frac{\frac{1}{n+4}\cdot\left(\frac{1+x}{1-x}\right)^{n+1}}{\frac{1}{n+3}\left(\frac{1+x}{1-x}\right)^n} =$$

$$= \lim_{n\to\infty} \frac{n+3}{n+4} \cdot \left(\frac{1+x}{1-x}\right) = \lim_{n\to\infty} \frac{\frac{n}{n}+\frac{3}{n}}{\frac{n}{n}+\frac{4}{n}} \cdot$$

$$\cdot \lim_{n\to\infty} \left(\frac{1+x}{1-x}\right) = \lim_{n\to\infty} \frac{1+\frac{3}{n}}{1+\frac{4}{n}} \cdot$$

$$\cdot \lim_{n\to\infty} \left(\frac{1+x}{1-x}\right) = \left(\frac{1+0}{1+0}\right) \lim_{n\to\infty} \frac{1+x}{1-x} =$$

$$= 1 \cdot \frac{1+x}{1-x}$$ ряд сходится при

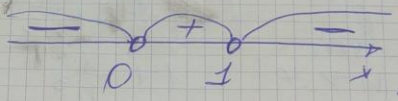
$\frac{1+x}{1-x} < 1$ $\quad \frac{1+x}{1-x} - 1 < 0$

$1+x < 1-x$ $\quad \frac{1+x-1+x}{1-x} < 0$

$1+x-1+x < 0$ $\quad \frac{2x}{1-x} < 0$

$2x < 0$

$x < 0$



$x \in (0;1)$.

$(0;1)$ область сходимости ряда.

№ 14.9

$$\sum_{n=1}^{\infty} \frac{n^3+1}{3^n (x-2)^n}$$

$$u_n = \frac{n^3+1}{3^n (x-2)^n} \qquad u_{n+1} = \frac{(n+1)^3+1}{3^{n+1}(x-2)^{n+1}}$$

$$\lim_{n\to\infty} \frac{u_{n+1}}{u_n} = \lim_{n\to\infty} \frac{\frac{(n+1)^3+1}{3^{n+1}(x-2)^{n+1}}}{\frac{n^3+1}{3^n (x-2)^n}} =$$

$$= \lim_{n\to\infty} \frac{(n+1)^3+1}{n^3+1} \cdot \frac{3^n (x-2)^n}{3^{n+1}(x-2)^{n+1}} =$$

$$= \lim_{n\to\infty} \frac{\frac{(n+1)^3}{n^3} + \frac{1}{n^3}}{\frac{n^3}{n^3} + \frac{1}{n^3}} \cdot \frac{1}{3} \cdot \frac{1}{(x-2)} =$$

$$= \lim_{n\to\infty} \frac{\left(1+\frac{1}{n}\right)^3 + \frac{1}{n^3}}{1+\frac{1}{n^3}} \cdot \frac{1}{3(x-2)} =$$

$$= \frac{(1+0)^3+0}{1+0} \cdot \frac{1}{3(x-2)} =$$

$$= \frac{1}{1} \cdot \frac{1}{3(x-2)} = \frac{1}{3(x-2)}$$

ряд сходится при

$$\frac{1}{3(x-2)} < 1$$

$$\frac{1}{3} < |x-2|$$

$$\begin{cases} x-2 > \frac{1}{3} \\ x-2 < -\frac{1}{3} \end{cases} \quad \begin{cases} x > \frac{7}{3} \\ x < \frac{5}{3} \end{cases}$$

$(-\infty; \frac{5}{3}) \cup (\frac{7}{3}; +\infty)$ – область сходимости.

19. 9 $\sum (x-1)\sin 5x$.

$$\sin x = 1 - \frac{x^2}{2!} + \frac{x^4}{4!} - \frac{x^6}{6!} + \frac{x^8}{8!} - \dots$$

$$\sin 5x = 1 - \frac{(5x)^2}{2!} + \frac{(5x)^4}{4!} - \frac{(5x)^6}{6!} + \frac{(5x)^8}{8!} - \dots$$

$$\sin 5x = 1 - \frac{5^2x^2}{2!} + \frac{5^4x^4}{4!} - \frac{5^6x^6}{6!} + \frac{5^8x^8}{8!} - \dots$$

$$(x-1)\cdot\sin 5x = (x-1)\cdot\left(1 - \frac{5^2x^2}{2!} + \frac{5^4x^4}{4!} - \frac{5^6x^6}{6!} + \frac{5^8x^8}{8!} - \dots\right) =$$

$$= x - 1 - \frac{5^2x^3}{2!} + \frac{5^2x^2}{2!} + \frac{5^4x^5}{4!} - \frac{5^4x^4}{4!} - \frac{5^6x^7}{6!} + \frac{5^6x^6}{6!} + \frac{5^8x^9}{8!} - \frac{5^8x^8}{8!} + \dots$$

20.9.

$$\int_0^{0,2} \sin 25x^2\,dx.$$

$$\sin x = x - \frac{x^3}{3!} + \frac{x^5}{5!} - \frac{x^7}{7!} + \frac{x^9}{9!} - \dots$$

$$\sin 25x^2 = 25x^2 - \frac{(25x^2)^3}{3!} + \frac{(25x^2)^5}{5!} - \frac{(25x^2)^7}{7!} + \frac{(25x^2)^9}{9!} - \dots$$

$$\sin 25x^2 = 25x^2 - \frac{25^3 x^6}{3!} + \frac{25^5 x^{10}}{5!} - \frac{25^7 x^{14}}{7!} + \frac{25^9 x^{18}}{9!} - \dots$$

$$\int_0^{0,2} \sin 25x^2\,dx = \int_0^{0,2} \left(25x^2 - \frac{25^3 x^6}{3!} + \frac{25^5 x^{10}}{5!} - \frac{25^7 x^{14}}{7!} + \frac{25^9 x^{18}}{9!} - \dots\right) dx =$$

$$= \left(25 \cdot \frac{x^3}{3} - \frac{25^3 x^7}{3! \cdot 7} + \frac{25^5 x^{11}}{5! \cdot 11} - \frac{25^7 \cdot x^{15}}{7! \cdot 15} + \frac{25^9 x^{19}}{9! \cdot 19}\right)\Bigg|_0^{0,2} =$$