1а) $\lim\limits_{x\to -8} \dfrac{\sqrt{1-x}-3}{\operatorname{arctg}(x+8)} = \left(\dfrac{0}{0}\right) =$

$= \left|\begin{array}{l}\text{Замена} \\ y = x-(-8) = x+8 \\ y \to 0 \quad x = y-8 \\ \text{при } x \to -8\end{array}\right| = \lim\limits_{y\to 0} \dfrac{\sqrt{1-(y-8)}-3}{\operatorname{arctg}(y-8+8)} =$

$= \lim\limits_{y\to 0} \dfrac{\sqrt{9-y}-3}{\operatorname{arctg} y} = \left|\begin{array}{l}\text{при } y\to 0 \\ \operatorname{arctg} y \sim y\end{array}\right| =$

$= \lim\limits_{y\to 0} \dfrac{(\sqrt{9-y}-3)\cdot(\sqrt{9-y}+3)}{y\cdot(\sqrt{9-y}+3)} =$

$= \lim\limits_{y\to 0} \dfrac{(\sqrt{9-y})^2 - 3^2}{y\cdot(\sqrt{9-y}+3)} =$

$= \lim\limits_{y\to 0} \dfrac{9-y+9}{y\cdot(\sqrt{9-y}+3)} =$

$= \lim\limits_{y\to 0} \dfrac{-y}{y\cdot(\sqrt{9-y}+3)} = \lim\limits_{y\to 0} \dfrac{-1}{\sqrt{9-y}+3} =$

$= \dfrac{-1}{\sqrt{9-0}+3} = \dfrac{-1}{3+3} = -\dfrac{1}{6}$

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15 $\lim\limits_{x\to 2} \dfrac{\sin(x-2)}{x^2-3x+2} =$

$= \left| \begin{array}{l} y = x-2 \\ x = y+2 \\ y \to 0 \text{ при } x \to 2 \end{array} \right| = \lim\limits_{y\to 0} \dfrac{\sin(y+2-2)}{(y+2)^2-3(y+2)+2} =$

$= \lim\limits_{y\to 0} \dfrac{\sin y}{y^2+4y+4-6y-6+2} =$

$= \lim\limits_{y\to 0} \dfrac{\sin y}{y^2-2y} =$

$= \left| \text{при } y \to 0 \ \sin y \sim y \right| =$

$= \lim\limits_{y\to 0} \dfrac{y}{y^2-2y} = \lim\limits_{y\to 0} \dfrac{y}{y\cdot(y-2)} =$

$= \lim\limits_{y\to 0} \dfrac{1}{y-2} = \dfrac{1}{0-2} = -\dfrac{1}{2}$

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Раздел 2
Задание 4

$$\begin{cases} x = t^2 + 1 \\ y = \frac{\ln t}{t} \end{cases} \qquad t = 1$$

$$y'_x = \frac{y'_t}{x'_t} = \frac{\left(\frac{\ln t}{t}\right)'_t}{(t^2+1)'_t} = \frac{\frac{(\ln t)' \cdot t - \ln t \cdot (t)'}{t^2}}{2t + 0} =$$

$$= \frac{\frac{1}{t} \cdot t - \ln t \cdot 1}{t^2 \cdot 2t} = \frac{1 - \ln t}{2t^3}$$

$$y'_x(t=1) = \frac{1 - \ln t}{2t^3} = \frac{1 - \ln 1}{2 \cdot 1^3} =$$

$$= \frac{1-0}{2} = \frac{1}{2}$$

$$y_0 = y(t=1) = \frac{\ln t}{t} = \frac{\ln 1}{1} = \frac{0}{1} = 0$$

$$x_0 = x(t=1) = t^2 + 1 = 1^2 + 1 = 1$$

Уравнение касательной

$$y = y_0 + y'(x_0) \cdot (x - x_0)$$

$$y = 0 + \frac{1}{2} \cdot (x - 1) = \frac{1}{2}x - \frac{1}{2}$$

Уравнение нормали

$$y = y_0 - \frac{1}{y'(x_0)} \cdot (x - x_0)$$

$$y = 0 - \frac{1}{(1/2)} \cdot (x - 1) = -2 \cdot (x - 1) = -2x + 1$$

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$$= \frac{\frac{1}{t} \cdot t - \ln t \cdot 1}{t^2 \cdot 2t} = \frac{1 - \ln t}{2t^3}$$

$$y'_x(t=1) = \frac{1 - \ln t}{2t^3} = \frac{1 - \ln 1}{2 \cdot 1^3} =$$

$$= \frac{1-0}{2} = \frac{1}{2}$$

$$y_0 = y(t=1) = \frac{\ln t}{t} = \frac{\ln 1}{1} = \frac{0}{1} = 0$$

$$x_0 = x(t=1) = t^2 + 1 = 1^2 + 1 = 1$$

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Уравнение нормали

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$$y = 0 - \frac{1}{(1/2)} \cdot (x - 1) = -2 \cdot (x - 1) = -2x + 1$$

Раздел 3

6) а) $\int (x-1)\cdot \ln x\, dx =$

$=$ | по частям
$\int u\,dv = u\cdot v - \int v\cdot du$
$u = \ln x \quad dv = (x-1)dx$
$du = u'dx = (\ln x)'dx = \frac{1}{x}dx$
$v = \int dv = \int (x-1)dx = \frac{x^2}{2} - x$

$= \ln x \cdot \left(\frac{x^2}{2} - x\right) - \int \left(\frac{x^2}{2} - x\right)\cdot \frac{1}{x}dx =$

$= \left(\frac{x^2}{2} - x\right)\cdot \ln x - \int \left(\frac{x}{2} - 1\right)dx =$

$= \left(\frac{x^2}{2} - x\right)\cdot \ln x - \left(\frac{x^2}{4} - x\right) + C$

6б) $\int \frac{dx}{(2x+3)^2} =$ | замена
$t = 2x+3$
$dt = (2x+3)'dx =$
$= (2\cdot 1 + 0)dx = 2\,dx$
$dx = \frac{dt}{2}$

$= \int \frac{\frac{dt}{2}}{t^2} = \frac{1}{2}\int t^{-2}dt = \frac{1}{2}\cdot \frac{t^{-2+1}}{-2+1} + C =$

$= \frac{1}{2}\cdot \frac{t^{-1}}{-1} + C = -\frac{1}{2t} + C = -\frac{1}{2\cdot(2x+3)} + C$

№6в) $\int_0^{2\pi} \frac{dx}{10+3\cos x} =$

$=$ | Универсальная тригонометрическая подстановка

$t = \operatorname{tg}\frac{x}{2} \qquad x = 2\operatorname{arctg} t$

$dx = \frac{2}{1+t^2}dt$

$\cos x = \frac{1-t^2}{1+t^2} \qquad \sin x = \frac{2t}{1+t^2}$

Если $x = 0$, то $t = \operatorname{tg}\frac{0}{2} = \operatorname{tg} 0 = 0$

Если $x = 2\pi$, то $t = \operatorname{tg}\frac{2\pi}{2} = \operatorname{tg}\pi = 0$

$$= \int_0^0 \frac{\frac{2}{1+t^2}dt}{10+3\cdot\frac{2t}{1+t^2}} = \int_0^0 \frac{\frac{2\,dt}{1+t^2}}{\frac{10+10t+6t^2}{1+t^2}} =$$

$$= \int_0^0 \frac{2\,dt}{10+10t+6t^2} = 0, \text{ т. к.}$$

$$\int_a^a f(x) = 0.$$

№2) $y = \frac{3}{(x-3)^2}$

$(x-3)^2 \neq 0 \Rightarrow x-3 \neq 0 \Rightarrow x \neq 3$

$D(x) = (-\infty; 3) \cup (3; +\infty)$

В точках области определения функция непрерывна

$\lim\limits_{x \to 3-0} y = \lim\limits_{x \to 3-0} \frac{3}{(x-3)^2} =$

$= \frac{3}{(3-0-3)^2} = \frac{3}{(-0)^2} = \frac{3}{+0} = +\infty$

$\lim\limits_{x \to 3+0} y = \lim\limits_{x \to 3+0} \frac{3}{(x-3)^2} =$

$= \frac{3}{(3+0-3)^2} = \frac{3}{(+0)^2} = \frac{3}{+0} = +\infty$

$x = 3$ - точка разрыва II рода является вертикальной асимптотой

№3) а) $y' = \left(1 - \sqrt[3]{x^2-2x}\right)' =$

$= 0 - \frac{1}{3} \cdot (x^2-2x)^{\frac{1}{3}-1} \cdot (x^2-2x)' =$

$= -\frac{1}{3} \cdot (x^2-2x)^{-\frac{2}{3}} \cdot (2x-2) =$

$= -\frac{1}{3} \cdot \frac{1}{(x^2-2x)^{2/3}} \cdot (2x-2) =$

$= -\frac{2x-2}{3 \cdot \sqrt[3]{(x^2-2x)^2}}$

б) $y' = \left(e^x \cdot (x^2+2x+2)\right)' =$

$= \boxed{(u \cdot v)' = u' \cdot v + u \cdot v'} =$

$= (e^x)' \cdot (x^2+2x+2) + e^x \cdot (x^2+2x+2)' =$

$= e^x \cdot (x^2+2x+2) + e^x \cdot (2x+2 \cdot 1+0) =$

$= e^x \cdot (x^2+2x+2+2x+2) =$

$= e^x \cdot (x^2+4x+4) = e^x \cdot (x+2)^2$

5) $y' = \left(\frac{3x^2+8x+16}{1-x}\right)' =$

$= \left[\left(\frac{u}{v}\right)' = \frac{u'v - uv'}{v^2}\right] =$

$= \frac{(3x^2+8x+16)' \cdot (1-x) - (3x^2+8x+16)\cdot(1-x)'}{(1-x)^2} =$

$= \frac{(6x+8)\cdot(1-x) - (3x^2+8x+16)\cdot(-1)}{(1-x)^2} =$

$= \frac{6x - 6x^2 + 8 - 8x + 3x^2 + 8x + 16}{(1-x)^2} =$

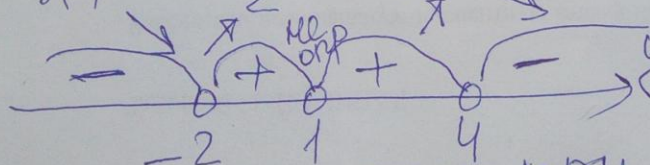
$= \frac{-3x^2 + 6x + 24}{(1-x)^2} = \frac{-3\cdot(x^2-2x-8)}{(1-x)^2}$

$y' = 0 \qquad \frac{-3\cdot(x^2-2x-8)}{(1-x)^2} = 0 \qquad x \neq 1$

$x^2 - 2x - 8 = 0$

$D = (-2)^2 - 4\cdot 1\cdot(-8) = 4 + 32 = 36$

$x_1 = \frac{2+6}{2} = 4 \qquad x_2 = \frac{2-6}{2} = -2$



$x_{min} = -2$

$x_{max} = 4$

Функция возрастает при $x \in (-2; 1) \cup (1; 4)$

Функция убывает при $x \in (-\infty; -2) \cup (4; +\infty)$

№7) начало

$y = \frac{1}{3}x\sqrt{x} - \sqrt{x} = 0$

$\sqrt{x} \cdot \left(\frac{1}{3}x - 1\right) = 0$

$\sqrt{x} = 0 \quad \frac{1}{3}x - 1 = 0$

$x = 0 \quad \frac{1}{3}x = 1$

$x = 3$

$$L = \int_{x_1}^{x_2} \sqrt{1 + (y')^2}\, dx$$

$y' = \left(\frac{1}{3}x\sqrt{x} - \sqrt{x}\right)' = \left(\frac{1}{3} \cdot x^{\frac{3}{2}} - x^{\frac{1}{2}}\right)' =$

$= \frac{1}{3} \cdot \frac{3}{2} \cdot x^{\frac{3}{2}-1} - \frac{1}{2} \cdot x^{\frac{1}{2}-1} =$

$= \frac{1}{2}x^{\frac{1}{2}} - \frac{1}{2} \cdot x^{-\frac{1}{2}} = \frac{1}{2}\sqrt{x} - \frac{1}{2\sqrt{x}}$

$L = \int_0^3 \sqrt{1 + \left(\frac{1}{2}\sqrt{x} - \frac{1}{2\sqrt{x}}\right)^2} =$

$= \int_0^3 \sqrt{1 + \frac{1}{4}x - 2 \cdot \frac{1}{2}\sqrt{x} \cdot \frac{1}{2\sqrt{x}} + \frac{1}{4x}}\, dx =$

$= \int_0^3 \sqrt{1 + \frac{1}{4}x - \frac{1}{2} + \frac{1}{4x}} = \int_0^3 \sqrt{\frac{4x + x^2 - 2x + 1}{4x}}\, dx$

Ну конец

$$= \int_0^3 \sqrt{\frac{x^2+2x+1}{4x}}\,dx = \int_0^3 \sqrt{\frac{(x+1)^2}{2^2 \cdot x}}\,dx =$$

$$= \int_0^3 \frac{x+1}{2\sqrt{x}}\,dx = \int_0^3 \left(\frac{x}{2\sqrt{x}} + \frac{1}{2\sqrt{x}}\right)dx =$$

$$= \int_0^3 \left(\frac{\sqrt{x}}{2} + \frac{1}{2\sqrt{x}}\right)dx =$$

$$= \left(\frac{\sqrt{x^3}}{2 \cdot \frac{3}{2}} + \frac{\sqrt{x}}{2 \cdot \frac{1}{2}}\right)\Bigg|_0^3 =$$

$$= \left(\frac{\sqrt{3^3}}{3} + \frac{\sqrt{3}}{1}\right) - \left(\frac{\sqrt{0^3}}{3} + \frac{\sqrt{0}}{1}\right) =$$

$$= \frac{3\sqrt{3}}{3} + \sqrt{3} - 0 = 2\sqrt{3}$$

№8 $\int_0^{+\infty} \frac{x}{x^2+4}\,dx =$

$= \lim\limits_{b\to+\infty} \int_0^b \frac{x\,dx}{x^2+4} =$

$=$ Замена
$t = x^2+4$
$dt = (x^2+4)'dx = 2x\,dx$
$x\,dx = \frac{dt}{2}$
Если $x=0$, то $t = 0^2+4 = 4$
Если $x=b$, то $t = b^2+4$

$= \lim\limits_{b\to+\infty} \int_4^{b^2+4} \frac{\frac{dt}{2}}{t} =$

$= \lim\limits_{b\to+\infty} \frac{1}{2}\ln|t|\Big|_4^{b^2+4} =$

$= \lim\limits_{b\to+\infty} \frac{1}{2}\ln|b^2+4) - \frac{1}{2}\ln|4| =$

$= \frac{1}{2}\ln(\infty^2+4) - \frac{1}{2}\ln 4 = \infty$

расходится